Decision Support for a Complex Spatial Scheduling Problem by a GIScience – Operations Research Approach: the case of VRPs with Pickup and Delivery and Time Windows

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Abstract

Scheduling is a well known problem from mathematics and has practical relevance especially for allocating limited resources in production processes. Thus, scheduling is generally regarded as having a temporal component. This paper elaborates on spatial scheduling problems which Vehicle Routing Problems (VRPs) are members of. In particular we focus on a complex class of VRPs that incorporates Pickup and Delivery as well as Time Windows. To provide decision support, this problem is formulated as a linear program. Additionally, time geography is employed to model the complexity of the spatio-temporal dimension inherent in VRPs. For simulation and optimization time geography is able to contribute by a filtering approach to identify feasible customers in the planning process of a vehicle route. This approach can be implemented in an optimization algorithm to provide decision support for VRPs.

1 Introduction

Scheduling describes a problem class where limited resources are committed to possible tasks with special consideration of the temporal sequence. In mathematics the Job Shop Scheduling is a well-known problem where the order of jobs on a set of machines is determined, given different jobs and machines to fulfill the tasks. The results of these problems are plans that define the temporal order of tasks in order to cope with the limited resources accordingly. Additionally, a given objective function has to be minimized – e.g. production costs or travel time.

Vehicle Routing Problems (VRPs) describe dispatching situations where a number of customers demand a good or service that are stored in a depot and delivered by a set of vehicles. Hence they are related to scheduling problems as they define the order of delivery processes while minimizing a given objective. These problems are spatial in nature and have relevance in everyday life. A classic example is the transportation of beer from the brewery to depots and from there to the pubs using trucks located at the depots.

Given the motivation above, this paper focuses on Vehicle Routing Problems (VRPs), which are well known problem classes in literature (e.g. TOTH & VIGO 2002). In general, VRPs are not solvable by computationally simple routines in short time – less than polynomial time (DASGUPTA ET AL. 2006). As they are defined by computational complexity theory as being NP-complete or NP-hard (KARP 1972), they require superpolynomial computational time. It can be noticed, that VRPs are spatial in nature, and thus have a relation to "real-world" problems that can be analyzed utilizing Geographical Information Science & Technology (GIS&T).

In order to solve Scheduling Problems, and VRPs in particular, a number of algorithms from Operations Research (OR) exist, that are categorized in three classes (TALBI 2009, PISINGER & ROPKE 2005, ROPKE 2005): Exact methods, Approximation algorithms and Heuristics. Exact methods require at least polynomial time to generate an optimal solution. Approximation algorithms are regarded as alternative. According to HOCHBAUM (1996) they provide a solution and an error guarantee, explaining that the obtained solution is at worst ε times more costly than the best solution. Heuristics are quick, with regard to computational speed, but do not provide a certain solution quality. Thus, the results may be suboptimal in comparison to exact algorithms. According to LAPORTE & SEMET (2002) Heuristics are categorized into three main classes: *Construction Heuristics*, *Two-phase Heuristics* and *Improvement Heuristics*. Basically, Construction Heuristics create a feasible solution, but do not optimize the result. Two-phase Heuristics decompose the VRP into the two underlying problems – clustering of customers and routing the vehicles. Subsequently, the two problems are solved one after the other – *either cluster-first, route-second* or *route-first, cluster-second* – with feedback between them. Improvement Heuristics try to improve a given solution based on a change of customers and/or vehicle routes.

In this paper the coupling of algorithms from OR with GIS&T theory is analyzed, which is necessary to model the spatio-temporal dimension. Our study is based on VRPs which are regarded as complex Scheduling Problems. In particular the VRP with Pickup and Delivery and Time Windows (VRPPDTW) is a specific VRP instance that adds several degrees of freedom to a standard VRP which increase the complexity. In this paper we present an approach to model VRPPDTWs with time geography. These findings are implemented in an Improvement Heuristic to solve VRPPDTWs accordingly. The implementation of the optimization Heuristic to optimize a VRPPDTW is not subject of this paper, as this is laid out in detail for the application area Wood Supply Chain in SCHOLZ & BARTELME (2010).

The organization of the paper is as follows: Chapter 2 highlights related scientific work done in this area. In chapter 3, VRPs are described, followed by a chapter on modeling VRPs with time geography. A basic sketch of a solution algorithm utilizing time geography is given in chapter 5. Chapter 6 elaborates on spatial decision support for the scheduling problem under investigation and chapter 7 concludes the paper and lists future research items.

2 Related Work

This section elaborates on the publications that are substantial for this particular work. This section discusses work published in the areas: Spatial Decision Support Systems (SDSSs), GIS&T and OR. Additionally, a considerable amount of progress has been achieved in the field of Forestry Decision Support Systems.

Foundations of SDSSs are the works of GORRY & SCOTT-MORTON (1971), who define basic decision types, necessary to distinguish computerized systems “helping” in a decision situation (DSS vs. Expert Systems). MALCZEWSKI (1999) defines the term Multi Criteria

Decision Analysis and describes how decision support is possible utilizing Geographical Information Systems (GISs). The state of the art of SDSSs is summarized in MALCZEWSKI (2006) where coupling techniques between GIS and decision support techniques are described too.

The VRP was first published by DANTZIG & RAMSER (1959). In order to get a general introduction in Graph Theory the book of JUNGNICKEL (2005) is appropriate. An overview of GIS and network analysis is given in FISCHER (2004). In this paper the VRP is described as a consisting of two subproblems: (a) finding an optimal assignment of customer orders to vehicles and (b) minimizing total travel cost by optimizing the vehicle routes. The description given in FISCHER (2004) utilizes the paper of FISHER & JAIKUMAR (1981). This publication introduces a solution methodology for VRPs based on the well-known *problems generalized assignment* and *Traveling Salesman Problem with Time Windows*, corresponding to the VRP subproblems (a) and (b) mentioned in FISCHER (2004). LAPORTE (1992) lists a number of VRP variants and solution algorithms including FISHER & JAIKUMAR (1981) approach. The behavior of the latter is described as iterating between solving the generalized assignment problem and the Traveling Salesman Problem with Time Windows repeatedly (LAPORTE 1992, p. 353), Hence, this approach is categorized as two-phase solution method – i.e. cluster-first, route-second method (LAPORTE & SEMET 2002). A contemporary overview on VRPs and their optimization techniques is given in TOTH & VIGO (2002) and in TOTH & VIGO (2002a). To solve VRPs, besides of exact and approximation techniques, heuristical techniques can be applied (e.g. TALBI 2009, BIANCHESINI & RIGHINI 2005, PISINGER & ROPKE 2005).

3 Vehicle Routing Problems

VRPs are problems where goods or services are distributed between customers and depots (TOTH & VIGO 2002). VRPs are at the intersection of the Bin Packing and the Traveling Salesman Problem, due to their nature of assigning a number of customers to a vehicle and developing optimized routes for each vehicle visiting all assigned customers. Hence, any VRP relies on the existence of vehicles located at one or more depots that perform the transportation process. The VRP results in a „plan“ for each vehicle, a route that starts and ends at a depot, satisfying the needs of the customers. Despite the fact that there are several variants of the VRP, TOTH & VIGO (2002) listed the following basic objects of a VRP:

- depot(s): start- and endpoint of vehicles; sometimes regarded as place where goods are stored;
- customer(s): entities that provide or demand goods or services; serviced by the vehicles;
- vehicle(s): navigate on the road network; transport goods and visit the customers; their routes start and end at the according depot;
- road network: defines the connections between customers and depots; must be defined as a connected graph $G=(V,E)$ with a set of vertices V and edges E;

The general case of the VRP is the Capacitated VRP (CVRP) that considers one depot, a set of customers and a defined number of vehicles located at the depot. The capacity of vehicles is limited. In the CVRP each customer has a demand of goods, which is formulated as a vector of demands d. Each single demand is non-negative and may not be split, whereas a depot has a demand of $d=0$. The CVRP results in a collection R of simple circuits $r=(v_0, v_1, \ldots, v_h, v_0)$, where r is corresponding to a vehicle route visiting a number of vertices v_n. Generally, any VRP seeks to construct routes such that the sum of the costs of all routes is minimal and that the following constraints are satisfied:

- each circuit visits the depot
- each customer is visited by exactly one circuit
- the sum of demands of the customers visited by one circuit not exceed the vehicle capacity

This problem is known to be NP-hard, which describes the fact that an exact solution cannot be computed in less than polynomial time in terms of the input size (KARP 1972).

In order to be able to handle additional problems that are related to the general CVRP a number of variations have been created, which are mentioned in TOTH AND VIGO (2002). In the course of this paper focus is given to the VRPPDTW. The concepts of Pickup and Delivery as well as Time Windows are described in detail in the next paragraphs.

The VRPPDTW describes a situation where each customer either provides or demands goods. Thus, the vehicles have to move goods from a pickup location to an appropriate delivery location. In addition, the customers need to be served within a defined time window, a concept described by e.g. DESAULNIERS ET AL. (2002). Hence, the VRPPDTW adds a number of additional constraints to the general CVRP. To formalize the constraints we consider a graph $G=(V,E,w)$ such that each customer is a vertex $v \in V$. Each customer i is associated with a time interval $[a_i, b_i]$ where a and b denote the start and end of the time window of customer i, within the service has to take place. Additionally, the service – i.e. the pick up or delivery process – has a duration of s_i time instants, which is denoted as service time. The quantity to be picked up or delivered is represented by the variables p_i and d_i, respectively. For each customer the vertex of origin O_i of the demanded and the destination vertex D_i of the goods to be picked up is defined. In order to model the temporal dimension the graph $G=(V,E,w)$ underlying the VRP uses weighted edges, where each edge $e=(v,u)$ is weighted with the travel time between vertices v and u. Additionally, the fastest and slowest possible speed for each edge is stored, which is necessary for spatio-temporal modeling described in chapter 4.

The objective of the VRPPDTW is to find a collection of circuits with minimum cost with respect to the following criteria:

- each circuit starts and ends at a depot
- each customer vertex is visited exactly by one circuit
- the load of a vehicle is non-negative and does not exceed the vehicle capacity
- for each customer i, the customer O_i has to be serviced before customer i, but within the same circuit

- for each customer *i*, the customer D_i has to be serviced after customer *i* but within the same circuit
- each customer *i* is serviced within the time window $[a_i, b_i]$
- during the service of any customer *i* the vehicle remains at the vertex *i* for s_i time instants

The VRPPDTW is, like the CVRP, NP-hard in the strong sense (KARP 1972). Hence, exact solution methods are very costly in terms of computational time. In order to overcome the limitations of exact algorithms, heuristics are employed and integrated in a decision support environment, which is described in chapter 5.

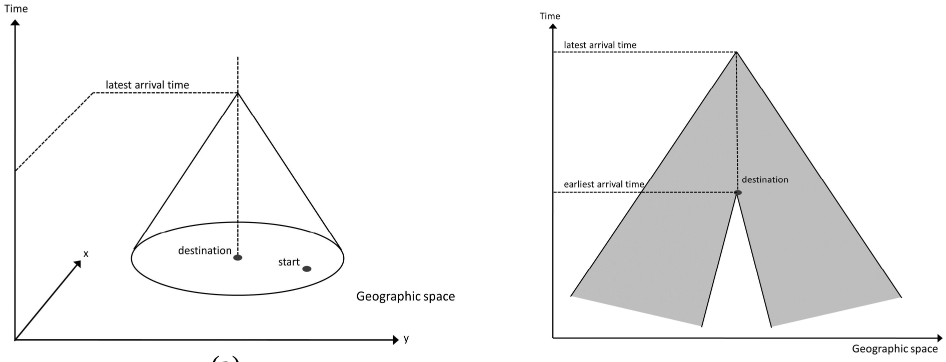
4 Time Geography and VRPs

Modeling and representing time and movement in Geographical Information Systems (GISs) is a "trend" in research, which is underpinned by publications by e.g. RAUBAL ET AL. (2007), KUIPERS ET AL. (2010). Based on the work of HÄGERSTRAND (1970) and LENNTORP (1976) this chapter aims to describe VRPs with the help of time geography. These considerations are later used for simulating and optimizing VRPs accordingly, which forms the basis for spatial decision support.

Time geography reflects the fact that a person or a resource is available at a certain location at a certain time. Each human is able to trade time against space and vice versa, and can thus be present at a specific location, which requires transportation (HÄGERSTRAND 1970). The representation of movements in space can be modeled with space-time paths. The slope of the path is an indicator of the travel speed. A vertical path denotes a stationary activity.

As time geography defines constraints of movements in space and time, the underlying space-time mechanics considers three constraints: capability, coupling and authority constraints (HÄGERSTRAND 1970). Basic physical restrictions are summarized under capability constraints. Coupling constraints describe the fact that two persons can only meet if they are at the same location at the same time. Authority constraints are used to model constraints of a certain domain, e.g. time windows for the service of customers. Space-time prisms (STPs) are two intersecting cones (LENNTORP 1976) in a space-time graph. This construct depicts the locations that can be visited, given a defined travel speed, a start and an end location. The interior of the STP is called potential path space, which represents all locations in space and time that can be reached. If the potential path space is projected to the geographic 2D space this results in the potential path area (MILLER 1991).

VRPPDTWs can be modeled and visualized using time geography, which gives insight in the complexity of such a scheduling problem. First we elaborate on the modeling approach for time window constraints at the customers, which is visualized in Fig. 1. In Fig. 1(a) a dl-cone is created that represents the potential locations of vehicles that are able to service the customer within the time window (RAUBAL ET AL. 2007). Due to the fact that dl-cones do not represent that any service must start at the start time of the time window a_i –waiting for the service time to start is not allowed – a cone is cut out of the dl-cone (see Fig. 1(b)), a complex dl-cone. The cone to be cut out denotes all locations that would reach the customer too early – i.e. before service time starts – even when driving at lowest allowed/possible speed. Hence, only those vehicles are considered to be at the customer within the time window whose location is within the cone. Following these principles any optimization algorithm is able to "filter" out those vehicles which are not of importance for a certain customer, which speeds up the optimization process at hand. Vice versa this approach is usable to select those customers that are of relevance for a certain vehicle or route.



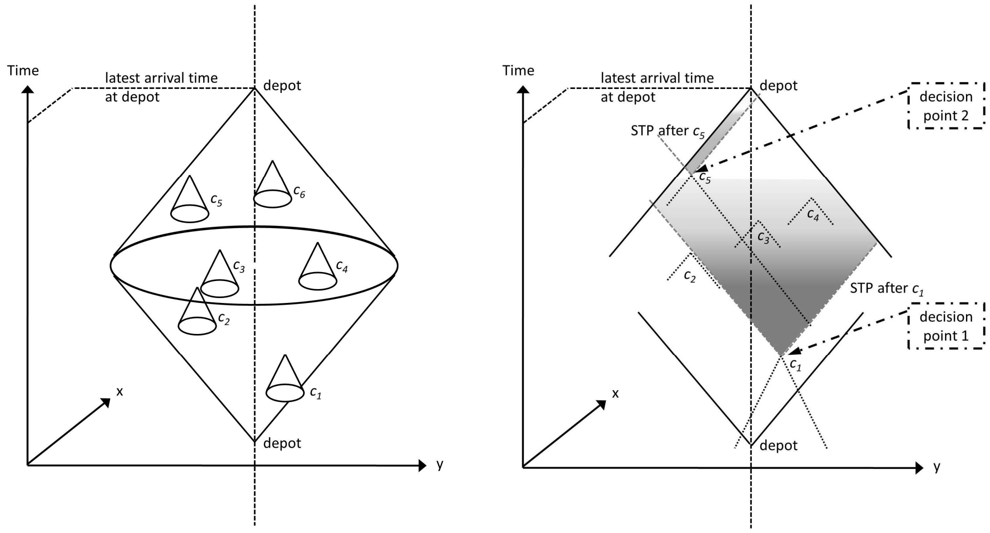
(a) (b)

Fig. 1: Dl-cone example in (a), from Raubal et al. (2007). An example of a complex dl-cone is given in (b).

Additionally, the potential path area of any vehicle starting and stopping at a certain depot is analyzed. While the potential path area spanning between the depot as start and endpoint has a certain size (see Fig. 2(a)), its size is decreased each time a decision is made to visit a certain customer. Considering VRPPDTWs, every decision has to be made with regard to the pairs of pickup and delivery customers. Once a pickup customer is visited, the according delivery customer has to be visited within the same circuit, which limits the possibilities of visiting other customers from a spatio-temporal perspective (see Fig. 2(b)). In Fig. 2(b) the customer c_1 is chosen as first serviced customer. Here we assume that the goods picked up at c_1 have to be delivered to c_5. Thus, at decision point 1 any optimization algorithm has to check if another customer can be inserted in the circuit before reaching c_5. From the figure it is clear that c_2 is not reachable within its time window, and c_2 is a candidate customer but c_5 cannot be serviced if c_2 is inserted in the circuit. At decision point 2 an optimization algorithm would have to check if any other customers could be serviced afterwards. Due to the fact that the potential path area is limited in its size – light grey colored area in Fig. 2(b) at the top of the STP – no additional customers are possible.

5 Basic Solution Algorithm

In order to solve the VRPPDTW, several heuristics are appropriate, which is mentioned in chapter 1 and chapter 2. In this paper we focus on one Improvement Heuristic – Adaptive Large Neighborhood Search (Pisinger & Ropke 2005, Ropke 2005) – and couple this Heuristic with time geography. This approach is able to help to increase the efficiency of the optimization process, by an intelligent selection process based on time geography, which will be described in this chapter.



(a) (b)

Fig. 2: The STP of a circuit starting and ending at a certain depot and six potential customers to be visited c_1-c_6, represented by complex dl-cones – simplified by dl-cones for visualization issues (a). In subfigure (b) the resulting STP after having visited c_1 is shown in dark grey, and if the circuit decides to serve c_5 after c_1 the resulting STP is shown in light grey.

Adaptive Large Neighborhood Search (ALNS) – depicted in Fig. 3 – starts from a given initial solution for any given VRPPDTW that is created using a Construction Heuristic and iteratively destroys and repairs the solution intelligently, in order to improve it. Thus, ALNS employs one of three available destroy heuristics to "ruin" a solution – remove customers from the solution – and subsequently repairs the solution using one of three repair methodologies. Both – destroy and repair – heuristics make use of spatial relations, which is laid out in SCHOLZ & BARTELME (2010) and SCHOLZ (2010). Especially for the repair process, time geography is of importance. These heuristics take the "destroyed" solution and try to include currently unassigned customers (not visited by any vehicle) into the solution at the position that results in the least increase of overall transport costs or which are hard to insert afterwards. This evaluation and selection process requires, regardless of the actual implementation, considerable computing power and time, given the numerous combinations of circuits, positions within the circuit and unassigned customers. Thus, time geography can help in pre-selecting only those customers that are feasible for a given insert position in a circuit, based on the considerations in chapter 4 (see Fig. 3). Hence, only these customers selected with the help of time geography are evaluated regarding their insert costs for a given insert position, which is reduces the evaluation effort and speeds up the optimization process. For details on ALNS procedure refer to Scholz & Bartelme (2010), PISINGER & ROPKE (2005), ROPKE (2005).

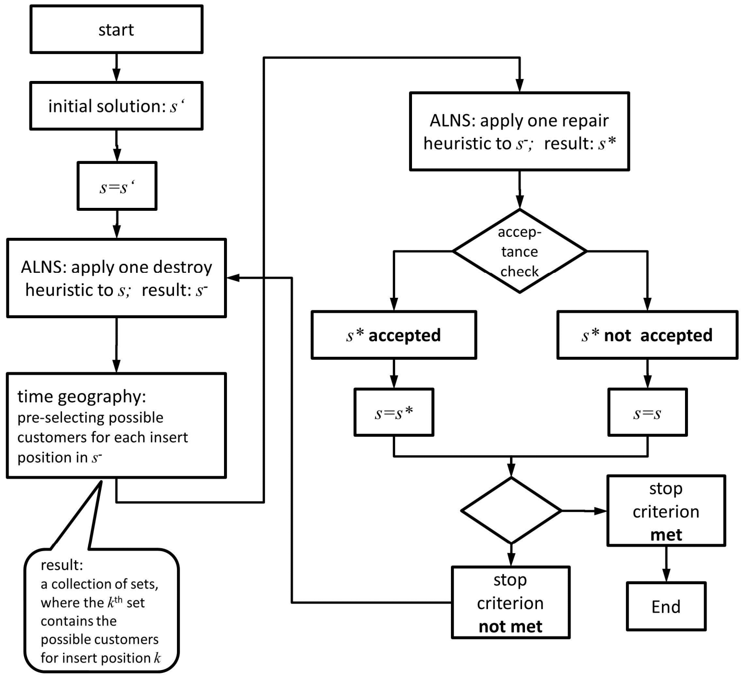


Fig. 3: Algorithmic sketch of ALNS with the pre-processing step using time geography before the application of a repair heuristic – based on SCHOLZ & BARTELME (2010).

6 Spatial Decision Support for Scheduling Problems

As scheduling problems are of practical nature, a number of “everyday life” situations – i.e. decisions –can be supported by software systems. In order to handle spatial problems MALCZEWSKI (1999) described the concept of Spatial Multi Criteria Decision Making, which forms the basis for SDSSs. MALCZEWSKI (1999) lists Model Base Management System, Model Base, Database Management System, Data Base, Dialog Generation and Management System as essential parts of any SDSS.

The Model Base and Model Base Management System contain the “intelligence” of any SDSS. For scheduling problems under review in this paper, a linear program can be defined that describes VRPPDTWs accordingly. Examples of such formulations are given in TOTH & VIGO (2002) and in SCHOLZ (2010). Based on a linear program in the model base, an optimization algorithm is able to generate solution alternatives and to evaluate them accordingly, as described in chapter 5. Subsequently, a decision matrix is created and its entries are ranked based on the evaluation. Due to the fact that a GIS is lacking such complex modeling methods (CHAKHAR & MARTEL 2003), optimization algorithms are coupled with GIS technology and inherited into a SDSS. This is necessary in order to benefit from the advantages both "worlds" have to offer.

7 Conclusion and Future Work

This paper elaborates on a specific problem class of spatial scheduling, namely VRPPDTW, and how spatial decision support can be achieved for this problem. The focus of this paper is on the description of VRPPDTWs as examples of complex scheduling tasks. The novelty of this paper is in the modeling of VRPs with the help of time geography. As time geography is not limited to the visualization of the complexity and spatio-temporal connections of VRPs, it is used for filtering purposes of feasible combinations of customer services and for the creation of vehicle routes within a VRP. The implementation of these considerations in an optimization heuristic facilitates spatial decision support for complex problems. Open research items in this context comprise a comparison of an optimization heuristic with and without time geography in terms of computational performance. In addition, the evaluation of other spatial scheduling problems, like harvest planning in forestry, or collaborative planning issues, relevant for practical applications. In addition, advancing the modeling in terms of time geography as well as the spatial enablement and application of optimization algorithms from OR can be listed as future research topics.

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